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A POINCARÉ–BENDIXSON TYPE THEOREM FOR HOLOMORPHIC VECTOR FIELDS

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INTRODUCTION

Let $\mathbf{Z}_1$ be a linear vector field on the two-dimensional complex space $\mathbf{C}^2$:

$$\mathbf{Z}_1 = \sum_{j=1}^{2} \lambda_j z_j \, \partial/\partial z_j\,, \quad \lambda_j \in \mathbf{C}\,, \quad \lambda_j \neq 0.$$

We have the following well-known

Fact ([1]). If λ_1/λ_2 does not belong to $\mathbf{R}_-$, the set of negative real numbers, then the three-dimensional unit sphere $S^3(1) = S^3(1:0)$ centered at the origin 0 in $\mathbf{C}^2$ is transverse to the foliation $\mathcal{F}(\mathbf{Z}_1)$ defined by the solutions of $\mathbf{Z}_1$.

If λ_1/λ_2 belongs to $\mathbf{R}_-$, $S^3(1)$ is not transverse to $\mathcal{F}(\mathbf{Z}_1)$.

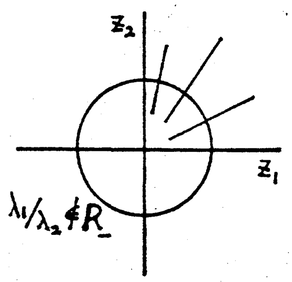

Fig. 1

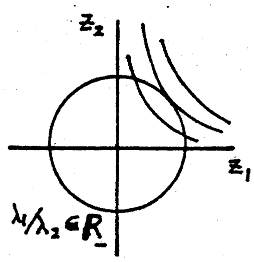

Fig. 2

We carry $S^3(1:0)$ to the sphere $S^3(1:(2,2))$ centered at the point $(2,2)$ in $\mathbf{C}^2$. Next we deform $S^3(1:(2,2))$ to $\tilde{S}^3(1:(2,2))$ as shown in Figures 5 and 6.

Intuitively it appears that $S^3(1:(2,2))$ and $\tilde{S}^3(1:(2,2))$ are not transverse to $\mathcal{F}(\mathbf{Z}_1)$. The above figures suggest to us a topological property of the transversality between spheres and holomorphic vector fields. This observation leads us to the following Poincaré–Hopf type theorem for holomorphic vector fields.

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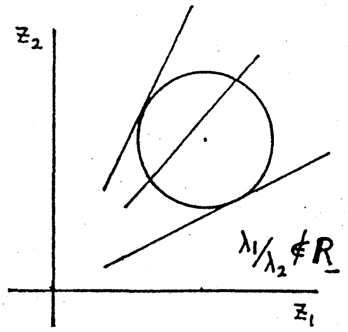

Fig. 3

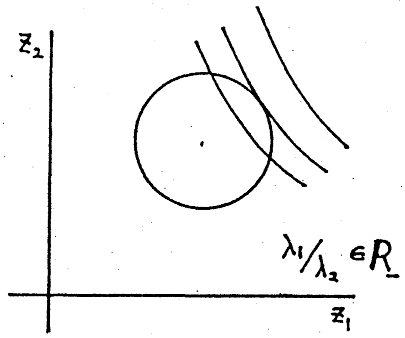

Fig. 4

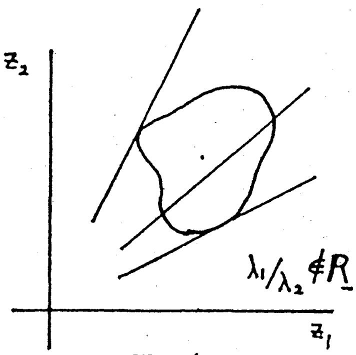

Fig. 5

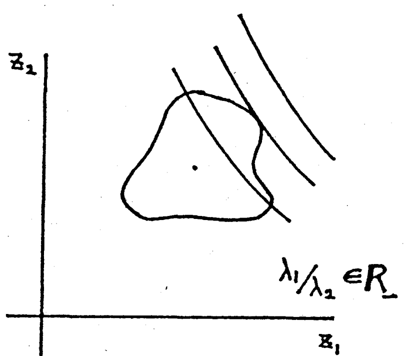

Fig. 6

Theorem 1. *Let M be a subset of $\mathbf{C}^n$, diffeomorphic to the $2n$–dimensional closed disk $\bar{D}^{2n}(1)$ consisting of all z in $\mathbf{C}^n$ with $\|z\| \leq 1$. We write $\mathcal{F}(\mathrm{Z})$ for the foliation defined by solutions of a holomorphic vector field Z in some neighborhood of M. If the boundary of M is transverse to $\mathcal{F}(\mathrm{Z})$, then Z has only one singular point, say p, in M. Furthermore, the index of Z at p is equal to one.*

From Theorem 1, we get an answer to the problem suggested by Figures 5 and 6.

Corollary 2. *Consider a linear vector field in $\mathbf{C}^n$: $\mathrm{Z} = \sum_{j=1}^{n} \lambda_j z_j\, \partial/\partial z_j$, $\lambda_j \in \mathbf{C}$, $\lambda_j \neq 0$. If a smooth imbedding φ of $(2n-1)$– sphere S^{2n-1} in $\mathbf{C}^n - \{0\}$ belongs to the zero element of the homotopy group $\pi_{2n-1}(\mathbf{C}^n - \{0\})$, then φ is not transverse to $\mathcal{F}(\mathrm{Z})$.*

Since the distance function for solutions of a holomorphic vector field Z with respect to the origin 0 is subharmonic, each solution of Z is unbounded except the singular set of Z. Therefore we have formulated a Poincaré–Bendixson type theorem for holomorphic vector fields.

Theorem 3. *Let M denote a subset of $\mathbf{C}^n$ holomorphic and diffeomorphic to the $2n$–dimensional closed disk $\bar{D}^{2n}(1)$. Let Z be a holomorphic vector field in some neighborhood of M. If the boundary ∂M of M is transverse to the foliation $\mathcal{F}(\mathrm{Z})$, then each solution of Z which crosses ∂M tends to the unique singular point p of Z in M, that is, p is in the closure* *of L. Further, the restriction $\mathcal{F}(\mathrm{Z})|_{M-\{p\}}$ of $\mathcal{F}(\mathrm{Z})$ to $M-\{p\}$ is C^ω-diffeomorphic to the foliation $\mathcal{F}(\mathrm{Z})|_{\partial M} \times (0, 1]$ of $M-\{p\}$, where $\mathcal{F}(\mathrm{Z})|_{\partial M}$ denotes the restriction of $\mathcal{F}(\mathrm{Z})$ to ∂M.*

Adrien Douady proved Theorem 3 in the case $n = 2$.

From Theorem 3 we get an affirmative answer to a special case of the Seifert conjecture.

Corollary 4. *Let* Z *be a holomorphic vector field in some neighborhood of* $\bar{D}^4(1) \subset \mathbf{C}^2$. *If the boundary* $\partial\bar{D}^4(1) = S^3(1)$ *is transverse to* $\mathcal{F}(\mathrm{Z})$, *then the restriction* $\mathcal{F}(\mathrm{Z})|_{S^3(1)}$ *to* S^3 *has at least one compact leaf.*

The author wishes to thank César Camacho for valuable discussions.

§1. Definition of transversality between manifolds and holomorphic vector fields

Let $\mathrm{Z} = \sum_{j=1}^n f_j(z)\,\partial/\partial z_j$ be a holomorphic vector field in the complex space $\mathbf{C}^n$ of dimension n. We identify $\mathbf{C}^n$ with the real space $\mathbf{R}^{2n}$ of dimension $2n$ by the natural correspondence. We have a real representation of Z:

$$\begin{aligned}
\mathrm{Z} &= \sum_{j=1}^n f_j(z)\,\partial/\partial z_j \\
&= \sum_{j=1}^n (g_j(x, y) + ih_j(x, y))\frac{1}{2}(\partial/\partial x_j - i\,\partial/\partial y_j) \\
&= \frac{1}{2}\left\{\left[\sum_{j=1}^n (g_j(x, y)\,\partial/\partial x_j + h_j(x, y)\,\partial/\partial y_j)\right]\right. \\
&\qquad\left. - i\left[\sum_{j=1}^n (-h_j(x, y)\,\partial/\partial x_j + g_j(x, y)\,\partial/\partial y_j)\right]\right\} \\
&= \frac{1}{2}(\mathrm{X} - i\mathrm{Y}),
\end{aligned} \tag{1.1}$$

where we set

$$\mathrm{X} = \sum_{j=1}^n (g_j(x, y)\,\partial/\partial x_j + h_j(x, y)\,\partial/\partial y_j) \tag{1.2}$$

and

$$\mathrm{Y} = \sum_{j=1} (-h_j(x, y)\,\partial/\partial x_j + g_j(x, y)\,\partial/\partial y_j). \tag{1.3}$$

Let $\mathbf{J}$ be the natural almost complex structure of $\mathbf{C}^n$. The vector fields $\mathbf{X}$ and $\mathbf{Y}$ satisfy the following equations:

$$\mathbf{JX} = \mathbf{Y}, \quad \mathbf{JY} = -\mathbf{X} \quad \text{and} \quad [\mathbf{X}, \mathbf{Y}] = 0. \tag{1.4}$$

Let N be a smooth manifold of dimension $2n-1$. We define below the transversality of a smooth map $\Phi : N \to \mathbf{C}^n$ to the foliation $\mathcal{F}(\mathbf{Z})$ determined by solutions of $\mathbf{Z}$.

Definition 1.1. We say that the map Φ is transverse to the foliation $\mathcal{F}(\mathbf{Z})$ or the holomorphic vector field $\mathbf{Z}$ if the following equation is satisfied for each point $p \in N$:

$$\Phi_*(T_pN) + \{\mathbf{X}, \mathbf{Y}\}_{\Phi(p)} = T_{\Phi(p)}\mathbf{R}^{2n},$$

where T_pN and $T_{\Phi(p)}\mathbf{R}^{2n}$ are the tangent space of N at p and the tangent space of $\mathbf{R}^{2n}$ at $\Phi(p)$ respectively, and $\{\mathbf{X}, \mathbf{Y}\}_{\Phi(p)}$ is the vector space generated by $\mathbf{X}_{\Phi(p)}$ and $\mathbf{Y}_{\Phi(p)}$. In particular, if N is a submanifold in $\mathbf{C}^n$, we say that N is transverse to $\mathcal{F}(\mathbf{Z})$.

For example consider the $(2n-1)$-dimensional sphere $S^{2n-1}(r)$, consisting of all $z \in \mathbf{C}^n$ with $\|z\| = r$. $S^{2n-1}(r)$ is tangent to $\mathcal{F}(\mathbf{Z})$ at $p \in S^{2n-1}(r)$ if and only if the following equation is satisfied at p:

$$\sum_{j=1}^{n} f_j(z)\bar{z}_j = \langle \mathbf{X}, \mathbf{N}\rangle - i\langle \mathbf{Y}, \mathbf{N}\rangle = 0, \tag{1.6}$$

where we denote by $\mathbf{N} = \sum_{j=1}^{n}(x_j\, \partial/\partial x_j + y_j\, \partial/\partial y_j)$ the usual normal vector field on $S^{2n-1}(r)$. We set $\Sigma = \{ z \in \mathbf{C}^n \mid \sum_{j=1}^{n} f_j(z)\bar{z}_j = 0 \}$ and say that Σ is the total contact set of spheres and $\mathcal{F}(\mathbf{Z})$. We denote by $R(z) = \sum_{j=1}^{n} |z_j|^2$ the distance function between $z \in \mathbf{C}^n$ and the origin 0 in $\mathbf{C}^n$. A critical point of the restriction $R|_L$ of R to a solution L of $\mathbf{Z}$ is a contact point of L and the sphere.

We will conclude this section by giving some examples of the contact set $\Sigma \cap S^{2n-1}(r)$ of $S^{2n-1}(r)$ and $\mathcal{F}(\mathbf{Z})$.

Example 1.2. Consider $\mathbf{Z} = z_1(2+z_1+z_2)\,\partial/\partial z_1 + z_2(1+z_1)\,\partial/\partial z_2$ defined in $\mathbf{C}^2$. The set $\mathrm{Sing}(\mathbf{Z})$ of singular points of $\mathbf{Z}$ consists of three points: $(0, 0)$, $(-2, 0)$ and $(-1, -1)$. Now $\mathrm{Sing}(\mathbf{Z}) \cap \bar{D}^4(1)$ consists of $(0, 0)$ only, where $\bar{D}^4(1)$ is the four-dimensional closed disk centered at the origin in $\mathbf{C}^2$ with radius 1. For any r, $0 < r \leq 1$, the contact set $S^3(r) \cap \Sigma$ is empty; that is, $S^3(r)$ is transverse to $\mathcal{F}(\mathbf{Z})$. Therefore, each solution of $\mathbf{Z}$ which crosses $S^3(1)$ tends to the origin in $\mathbf{C}^2$.

Example 1.3. Let a be a complex number different from zero. Define Z on C^2 by $\mathrm{Z} = (2z_1 + az_2^2)\,\partial/\partial z_1 + z_2\,\partial/\partial z_2$. We mention here that one can find in [3] one of the normal forms of holomorphic vector fields in C^2:

$$\widetilde{\mathrm{Z}} = (\lambda_1 z_1 + a z_2^n)\,\partial/\partial z_1 + \lambda_2 z_2\,\partial/\partial z_2\,, \qquad \lambda_1 = n\lambda_2.$$

The singular set $\mathrm{Sing}(\mathrm{Z})$ consists of a single point $(0\,,\,0)$. There exists a number $r_0 > 0$ such that
(i) if $0 < r < r_0$, $\Sigma \cap S^3(r)$ is empty:
(ii) if $r = r_0$, $\Sigma \cap S^3(r_0)$ is diffeomorphic to the circle S^1;
(iii) if $r_0 < r$, $\Sigma \cap S^3(r)$ is diffeomorphic to the disjoint union $S^1 \coprod S^1$ of two copies of the circle S^1.

In the case (ii), the circle $\Sigma \cap S^3(r_0)$ consists of degenerate critical points. If L_p is the solution of Z passing through $p \in \Sigma \cap S^3(r_0)$, then $L_p \cap \Sigma$ is a singleton set $\{p\}$.

In the case (iii), one circle of $\Sigma \cap S^3(r)$ consists of minimal points and the other consists of saddle points. In particular, for $p \in \Sigma \cap S^3(r)$ the set $L_p \cap \Sigma$ consists of two points p and q, $p \neq q$. More precisely, one of these two points is a saddle point of $R|_{L_p}$ and the other a minimal point of $R|_{L_p}$.

Example 1.4. One finds in [4] the following example of a one-form ω on C^2: $\omega = z_2(1 - i - z_1 z_2)\,dz_1 - z_1(1 + i - z_1 z_2)\,dz_2$. We consider here $\mathrm{Z} = z_1(1 + i - z_1 z_2)\,\partial/\partial z_1 + z_2(1 - i - z_1 z_2)\,\partial/\partial z_2$ on C^2. The singular set $\mathrm{Sing}(\mathrm{Z})$ consists of a single point, namely $(0\,,\,0)$. If $0 < r < \sqrt{2}$, $\Sigma \cap S^3(r)$ is empty. If $r = \sqrt{2}$, $\Sigma \cap S^3(\sqrt{2})$ is diffeomorphic to the circle S^1. Indeed $\Sigma \cap S^3(\sqrt{2})$ belongs to the solution $z_1 z_2 = 1$ of Z. If $r > \sqrt{2}$, $\Sigma \cap S^3(r)$ is diffeomorphic to the disjoint union $S^1 \coprod S^1$ of two copies of the circle S^1, and consists of saddle points.

§2. Proof of Theorem 1

In this section we shall use the same notation as in the previous sections.

First, we note that the following property of analytic sets in C^n: the set of singular points of Z in M consists of isolated finite points. Since the boundary ∂M of M is transverse to $\mathcal{F}(\mathrm{Z})$, there exists a smooth vector field ξ in some neighborhood of ∂M such that

(i) ξ is represented by $a\mathrm{X} + b\mathrm{Y} \neq 0$, where a and b are smooth functions defined in some neighborhood of ∂M;
(ii) ξ is required to point outward at each point of ∂M.

We obtain a smooth map $(a\,,\,b)$ of some neighborhood of ∂M to $\mathrm{R}^2 - \{0\}$. When $n \geq 2$ using obstruction theory (see [9]), we can extend the map $(a\,,\,b)$ to a smooth map $(\alpha\,,\,\beta)$ of some neighborhood of M to $\mathrm{R}^2 - \{0\}$ such that the restriction of $(\alpha\,,\,\beta)$ to some neighborhood of ∂M is the map $(a\,,\,b)$.

There should be no confusion if we use ξ for the extended smooth vector field $\xi = \alpha\mathbf{X} + \beta\mathbf{Y}$. By the definition of ξ on a neighborhood of M, the set $\mathrm{Sing}(\mathbf{Z})$ of the singular points of $\mathbf{Z}$ coincides with that of ξ.

In order to calculate the index of ξ at $p \in \mathrm{Sing}(\mathbf{Z})$, we may think of the vector field ξ as a map $\xi : M \to \mathbf{R}^{2n}$. Similarly we may think of the holomorphic vector field $\mathbf{Z}$ as a map $\mathbf{Z} : M \subset \mathbf{C}^n \to \mathbf{C}^n$ or as a map $\mathbf{Z} : M \subset \mathbf{R}^{2n} \to \mathbf{R}^{2n}$. We say that the vector field $\mathbf{Z}$ is non-degenerate at $p \in \mathrm{Sing}(\mathbf{Z})$ if the Jacobian $\det(D(\mathbf{Z})(p))$ of $\mathbf{Z}$ at p is different from zero. By a direct calculation we obtain the following:

$$\begin{aligned} \det(D(\xi)(p)) &= \det\begin{pmatrix} \alpha(p)I_n & -\beta(p)I_n \\ \beta(p)I_n & \alpha(p)I_n \end{pmatrix} \det(D(\mathbf{Z})(p)) \\ &= |\det((\alpha(p) + i\beta(p))I_n)|^2 \left|\det\left(\frac{\partial g_j}{\partial x_k}(p) + i\frac{\partial g_j}{\partial y_k}(p)\right)\right|^2, \end{aligned} \tag{2.1}$$

where $\det A$ denotes the determinant of a matrix A and I_n is the identity matrix of $GL(n, \mathbf{R})$. In particular, since $\det(D(\mathbf{Z})(p))$ is positive at a non-degenerate singular point $p \in \mathrm{Sing}(\mathbf{Z})$, the index of ξ at p is one (see [6]).

In order to calculate the index of ξ at a degenerate singular point $p \in \mathrm{Sing}(\mathbf{Z})$, we recall the following

Proper mapping theorem ([5]). Let $F : \mathbf{C}^n \to \mathbf{C}^n$ be a holomorphic map such that $F(0)$ is equal to 0. Assume that 0 is an isolated point in $F^{-1}(0)$ and $\det(D(F)(0))$ is 0. Then there exists a number $\epsilon > 0$ together with a neighborhood W of 0 such that $F|_W : W \to \Delta(0 : \epsilon) = \{ z \in \mathbf{C}^n |\ \|z\| < \epsilon \}$ is surjective.

Using the proper mapping theorem we find a sufficiently small number $\epsilon > 0$ and a neighborhood W of $p \in \mathrm{Sing}(\mathbf{Z})$ such that $W \cap \mathrm{Sing}(\mathbf{Z})$ is a singleton set. Since there exist regular values y of $\mathbf{Z}$ in $\Delta(0 : \epsilon)$, by (2.1), we may select a regular value y of ξ in $\Delta(0 : \epsilon_1) = \{ y \in \mathbf{R}^{2n} |\ \|y\| < \epsilon_1 \}$, $0 < \epsilon_1 < \epsilon$. The set $N_1 = \xi^{-1}(\bar{\Delta}(0 : \epsilon_1)) \cap W$ is compact. We then choose a compact set N with $W \supset N \supset N_1$ and a smooth function λ which takes on the value one at $x \in N_1$ and zero at $x \notin N$. Define $\tilde{\xi}$ by $\tilde{\xi}(x) = \xi(x) - \lambda(x)y$. Then $\tilde{\xi}$ is different from zero at each point $x \in N - N_1$; hence $\tilde{\xi}^{-1}(0) \cap W$ is compact and each point $\tilde{p} \in \tilde{\xi}^{-1}(0) \cap W$ is non-degenerate. Now we are ready to calculate the index of the vector field ξ at a degenerate point $p \in \mathrm{Sing}(\mathbf{Z})$:

$$\begin{aligned} \mathrm{index}_p\, \xi &= \sum_{\tilde{p} \in \tilde{\xi}^{-1}(0) \cap W} \mathrm{index}_{\tilde{p}}\, \tilde{\xi} \\ &= \text{the number of elements of } \tilde{\xi}^{-1}(0) \cap W \geq 1, \end{aligned} \tag{2.2}$$

where $\mathrm{index}_p\, \xi$ denotes the index of ξ at p.

On the other hand, by the Poincaré–Hopf theorem we have the following:

$$1 = \chi(M) = \sum_{p \in \mathrm{Sing}(\mathrm{Z}) \cap M} \mathrm{index}_p \, \xi \,, \tag{2.3}$$

where $\chi(M)$ denotes the Euler number of M. From (2.2) and (2.3) we conclude that the number of elements of $\mathrm{Sing}(\mathrm{Z})$ in M is one. This completes the proof of Theorem 1.

§3. Proof of Theorem 3

We continue to use the same notation.

Since M is holomorphic, diffeomorphic to the $2n$–dimensional closed disk $\bar{D}^{2n}(1)$, we give a proof of Theorem 3 for $\bar{D}^{2n}(1)$. Using a Möbius transformation, we can assume that the sole singular point of Z in $\bar{D}^{2n}(1)$ is the origin 0. We define a function F in some neighborhood of $\bar{D}^{2n}$ minus the origin 0 by

$$F(z) = \frac{\sum_{j=1}^{n} f_j(z)\bar{z}_j}{\sum_{j=1}^{n} |z_j|^2}.$$

Since the boundary $S^{2n-1}(1)$ of $\bar{D}^{2n}(1)$ is transverse to $\mathcal{F}(\mathrm{Z})$, the restriction $F|_{S^{2n-1}(1)}$ of F to $S^{2n-1}(1)$ takes on the values in $\mathrm{C} - \{0\}$. Consider a complex line l_z through a point $z \in S^{2n-1}(1)$: $l_z = \{ tz \in \mathrm{C}^n \,|\, t \in \mathrm{C} \}$. We define a holomorphic function $\tilde{F}(t : z)$ in some neighborhood of $\bar{D}^2(1 : 0) = \{ t \in \mathrm{C} \,|\, |t| \leq 1 \}$ by

$$\tilde{F}(t : z) = \begin{cases} \dfrac{\sum_{j=1}^{n} f_j(tz)\bar{t}\bar{z}_j}{t\bar{t}}, & \text{if } t \neq 0 \\ \displaystyle\sum_{j\,,\,k=1}^{n} \frac{\partial f_j}{\partial z_k}(0) z_k \bar{z}_j \,, & \text{if } t = 0. \end{cases}$$

Then the degree of $\tilde{F}|_{|t|=1}$ is zero, because $F|_{S^{2n-1}(1)}$ is homotopic to a constant map. Hence, for any $z \in S^{2n-1}(1)$, $\tilde{F}(t : z)$ is not zero; that is, the only element of $\Sigma \cap \bar{D}^{2n}(1)$ is the origin 0 in C^n. In other words, $S^{2n-1}(r)$, $0 < r \leq 1$, are transverse to $\mathcal{F}(\mathrm{Z})$. Let $\tilde{\mathrm{N}} \in T\mathcal{F}(\mathrm{Z})$ be the vector field of the projection of N to $T\mathcal{F}(\mathrm{Z})$. The set of singular points of $\tilde{\mathrm{N}}$ in $\bar{D}^{2n}(1)$ is the singleton set $\{0\}$ in C^n. Then each solution of Z which crosses $S^{2n-1}(1)$ tends to 0 along the orbit of $\tilde{\mathrm{N}}$. Furthermore, the restricted foliation $\mathcal{F}(\mathrm{Z})|_{S^{2n-1}(r)}$ of $S^{2n-1}(r)$ is C^{ω}-diffeomorphic to the foliation $\mathcal{F}(\mathrm{Z})|_{S^{2n-1}(1)}$ of $S^{2n-1}(1)$ by the correspondence along orbits of $\tilde{\mathrm{N}}$. This completes the proof of Theorem 3.

§4. A special case of Seifert conjecture

The notation used in the Introduction, §1 and §3 carries over in the present section.

We first recall the Seifert conjecture. Consider the vector field $\mathrm{e} = z_1\,\partial/\partial z_1 + z_2\,\partial/\partial z_2$ on $\mathbf{C}^2$. All leaves of the restricted foliation $\mathcal{F}(\mathrm{e})|_{S^3(1)}$ of $S^3(1)$ are fibres of the Hopf fibration $S^3 \to S^2$. On the other hand, consider the vector field $\mathrm{e}_\epsilon = (z_1 + \epsilon z_2)\,\partial/\partial z_1 + z_2\,\partial/\partial z_2$, where the number ϵ is sufficiently small. Then the restricted foliation $\mathcal{F}(\mathrm{e}_\epsilon)|_{S^3(1)}$ of $S^3(1)$ has one closed orbit $|z_1| = 1$ but all other leaves are diffeomorphic to $\mathbf{R}^1$. In [8] H. Seifert proved the following

Theorem (H. Seifert). A continuous vector field on the three-sphere which differs sufficiently little from $\mathcal{F}(\mathrm{e})|_{S^3(1)}$ and which sends through every point exactly one integral curve, has at least one closed integral curve.

The Seifert conjecture says "every non-singular vector field on the three-dimensional sphere S^3 has a closed integral curve".

In [7] Paul Schweitzer constructed a counterexample to the Seifert conjecture: There exists a non-singular C^1 vector field on S^3 which has no closed integral curves.

In this section we investigate a certain property of a non-singular vector field on S^3 induced by a holomorphic vector field in some neighborhood of $\bar{D}^4(1)$ which is transverse to $S^3(1)$. This will prove Corollary 4.

Proof of Corollary 4. Using a Möbius transformation, we can assume that the only singular point of $\mathbf{Z}$ in $\bar{D}^4(1)$ is the origin. First, we note that the existence of a separatrix of $\mathbf{Z}$ at 0 was proved by C. Camacho and P. Sad [2]. Let L be a separatrix of $\mathbf{Z}$ at 0. There is a sufficiently small number $\epsilon > 0$ together with a holomorphic function f defined in $D^4(\epsilon)$ such that $D^4(\epsilon) \cap \bar{L} = \{f = 0\}$. Then for each ϵ_1, $0 < \epsilon_1 < \epsilon$, $S^3(\epsilon_1) \cap L$ is a circle. Since $\mathcal{F}(\mathbf{F})|_{S^3(\epsilon_1)}$ is C^ω-diffeomorphic to $\mathcal{F}(\mathbf{F})|_{S^3(1)}$, the latter has at least one compact leaf. This completes the proof of Corollary 4.

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